SOLUTIONS TO MATH PULSE CHECK PROBLEMS

Averages p. 2

1. Mike's grade is based on five tests. On four of the tests, he scores 80, 85, 86, and 84. What must he score on his final test in order to make his average on all five tests 85?

$$\frac{80+85+84+86+x}{5}=85$$
$$335+x=425$$
$$x=90$$

2. The average of three consecutive integers 4. What is the smallest possible value of one of these integers?

   The average of any three consecutive integers will always be the middle number. For example, the average of 5, 6, and 7 is 6. So, if 4 is the average, it's at the middle, making the numbers 3, 4, and 5. **The answer is thus 3.**

Weighted Averages p. 3

1. A stockbroker invests 30% of his portfolio in a stock that earns him $5 and 70% of his portfolio into a stock in which he loses $2. What is the stockbroker's average return, in dollars?

   Weighted Avg = $.3(5)+.7(-2)=0.10$

Sequences and Series p. 4

1. The first term in a sequence is 40 and each term thereafter is obtained by dividing the previous term by 2. For example, the second term is 40/2 = 20. What is the value of the first term in the sequence that is not an integer?

   The sequences goes 40, 20, 10, 5, and then **2.5.**

2. $-1,3,-9,27...$

   The first term in the sequence above is -1. Each subsequent term in the sequence is obtained by multiplying the previous term by -3. What is the average of the 6th and 7th terms?

   The full sequence goes $-1,3,-9,27,-81,243,-729$

   The average of 243 and -729 is -243.

3. $3,5,6,7,8,3,5,6,7,8...$

   The numbers 3,5,6,7, and 8 repeat indefinitely in the sequence above. What is the sum of the 104th and 200th terms?

$$\frac{104}{5} = 20.8$$
$$.8x5 = \text{R } 4$$

When the remainder is 0, the sequence stops on 8. Since the remainder is 0, we need to go four numbers over to 7.

$\frac{200}{5} = 40$ There's no remainder, making the 200th term 8.

Answer: 7 + 8 = 15.

Funky Symbols page 5

1. $x \,\&\, y = 2xy + 4$. What is 3 & -5?

$$x \,\&\, y = 2xy + 4$$
$$3 \,\&\, -5 = 2(3)(-5) + 4 = -26$$

2. $a @ b = 2ab + 5a$. If 4 @ x = 84, what is the value of x?

$$a @ b = 2ab + 5a$$
$$4 @ x = 2(4)(x) + 5(4) = 8x + 20$$

$$8x + 20 = 84$$
$$8x = 64$$
$$x = 8$$

Exponents p. 7

1. $9^{4x} = 27^{3x+3}$. What is the value of x?

$$9^{4x} = 27^{3x+3}$$
$$\left(3^2\right)^{4x} = \left(3^3\right)^{3x+3}$$
$$3^{8x} = 3^{9x+9}$$
$$8x = 9x + 9$$
$$-9 = x$$

2. $\left(x^5\right)^{\frac{1}{2}} = \left(x^2\right)\left(x^{2a}\right)$. What is the value of a?

$$\left(x^5\right)^{\frac{1}{2}} = \left(x^2\right)\left(x^{2a}\right)$$
$$x^{\frac{5}{2}} = x^{2+2a}$$
$$\frac{5}{2} = 2 + 2a$$
$$\frac{1}{2} = 2a$$
$$\frac{1}{4} = a$$

3. $\left(27x\right)^{2/3} = 9x^{2a}$. What is the value of a?

$$\left(27x\right)^{2/3} = 9x^{2a}$$
$$27^{\frac{2}{3}}x^{\frac{2}{3}} = 9x^{2a}$$
$$9x^{\frac{2}{3}} = 9x^{2a}$$
$$x^{\frac{2}{3}} = x^{2a}$$
$$\frac{2}{3} = 2a$$
$$\frac{1}{3} = a$$

Word Problems p. 8

1. Six less than twice the square root of an integer is -1. What is the integer?

$$2\sqrt{n} - 6 = -1$$
$$2\sqrt{n} = 5$$
$$\sqrt{n} = \frac{5}{2}$$
$$n = \frac{25}{4}$$

2. Two more than five times a number is 17. What is the number?

$$2 + 5n = 17$$
$$5n = 15$$
$$n = 3$$

Direct and Indirect Variation p. 11

1. y is inversely proportional to x. If y is 40 when x is 2, what is y when x is 20?

$$xy = k$$
$$(2)(40) = k$$
$$80 = k$$

$$xy = 80$$
$$(20)y = 80$$
$$y = 4$$

2. x varies directly as the cube of y. If x is 4 when y is 2, what is x when y is 3?

$$x = ky^3$$
$$4 = k\left(2^3\right)$$
$$4 = 8k$$
$$\frac{1}{2} = k$$

$$x = \frac{1}{2}y^3$$
$$x = \frac{1}{2}3^3 = \frac{27}{2}$$

Greatest Possible Value p. 11

1. $(x+y)^2 = 25$. What is the greatest possible value of $x + y - (x + y)$?

$$(x+y)^2 = 25$$
$$x + y = 5 \ OR \ -5$$

$$x + y - (x + y) = 5 - (-5) = 10$$

Exponential Growth p.13

1. The initial value of a car is \$2000. If the value of the car decreases by 15% each year, write a formula that expresses the cost of the car after n years.

$$y = 2000(.85)^n$$

2. The initial value of a guitar is \$1000. If the value of the guitar increases by 10% each year, write a formula that expresses the value of the guitar after n years.

$$y = 1000(1.1)^n$$

Difference of Two Squares p. 13

1. If $x + y = 10$ and $x - y = 2$, what is $x^2 - y^2$?

$$x^2 - y^2 = (x + y)(x - y)$$
$$x^2 - y^2 = (10)(2) = 20$$

2. If $x^2 - y^2 = 80$ and $x - y = 8$, what is the value of $x^2 + 2xy + y^2$?

$$x^2 - y^2 = (x + y)(x - y)$$
$$80 = (x + y)(8)$$
$$10 = (x + y)$$

$$x^2 + 2xy + y^2 = (x + y)(x + y) = (10)(10) = 100$$

Absolute Value p. 15

1. A certain factory requires that its product weight x be between 20 and 60 pounds. Which of the following equations represents all possible values of x?

A) $|x - 40| \le 10$
B) $|x - 40| \le 20$
C) $|x - 20| \le 20$
D) $|x - 20| \le 30$
E) $|x - 30| \le 10$

2. $|2 - 2x| = 16$
$|2 - 2y| = 8$

What is the largest possible value of x - y?

$$|2 - 2x| = 16$$
$$2 - 2x = 16 \quad \text{OR} \quad 2 - 2x = -16$$
$$x = -7 \qquad\qquad x = 9$$

$$|2 - 2y| = 8$$
$$2 - 2y = 8 \quad \text{OR} \quad 2 - 2y = -8$$
$$y = -3 \qquad\qquad y = 5$$

$$x - y = 9 - (-3) = 12$$

3. If $|6x+2|<8$, which of the following is a possible value of x?

A) -3
B) -1
C) 3
D) 5
E) 7

4. If $|3-2x|<5$, what is one possible value of x?

$$|3-2x|<5$$
$$-5<3-2x<5$$
$$-8<-2x<2$$
$$4>x>-1$$
$$-1<x<4$$

So anything between -1 and 4 (but not including) is fine.

Equations of Lines p. 17, 18

1. If (a,4) is a point on the graph of y = -2x+15, what is the value of a?

$$y=-2x+15$$
$$4=-2a+15$$
$$\frac{11}{2}=a$$

2. What is the equation of the graph that passes through the origin and is perpendicular to the line $x+2y=12$?

$$x+2y=12$$
$$2y=-x+12$$
$$y=-\frac{1}{2}x+6$$

If the line is perpendicular, the slope will be the opposite reciprocal, or 2. Since the line goes through the origin and has a slope of 2, the equation $y=mx+b$ becomes $y=2x+0$ or just $y=2x$.

3. At what y value do the graphs of $y = 2x + 4$ and $y = \frac{1}{2}x$ intersect?

$$2x + 4 = \frac{1}{2}x$$
$$\frac{3}{2}x = -4$$
$$x = -\frac{8}{3}$$

$$y = \frac{1}{2}x = \frac{1}{2}\left(-\frac{8}{3}\right) = -\frac{4}{3}$$

4. $y = ax + 6$ and $y = 6x + 5$ intersect at the **point (b,7).** What is the value of a?

Note that there's a typo in the original notes. Using the point (b,7), we solve:

$$y = 6x + 5$$
$$7 = 6b + 5$$
$$\frac{1}{3} = b$$

Our point is now $\left(\frac{1}{3}, 7\right)$. Plugging that into the second equation, we get:

$$y = ax + 6$$
$$7 = a\left(\frac{1}{3}\right) + 6$$
$$1 = \frac{a}{3}$$
$$3 = a$$

In Terms Of p. 18

1. $2x + y = 2(4x + y)$. What is x in terms of y?

$$2x + y = 2(4x + y)$$
$$2x + y = 8x + 2y$$
$$-6x = y$$
$$x = -\frac{y}{6}$$

2. $\frac{3x+6y}{6}=\frac{2x+3}{3}$. What is x in terms of y?

$$\frac{3x+6y}{6}=\frac{2x+3}{3}$$
$$3(3x+6y)=6(2x+3)$$
$$9x+18y=12x+18$$
$$-3x=-18y+18$$
$$x=6y-6$$

Deriving Formulas/Plugging Numbers p. 21 - 23

1. C
2. B
3. A
4. C
5. BAD PROBLEM
6. B
7. E (if you picked D, you made a silly mistake)

Algebra Manipulations p. 25

1. If $2x=30-2y$, what is $x+y$?

$$2x=30-2y$$
$$2x+2y=30$$
$$x+y=15$$

2. If $x^{a+5}=40$ and $x^5=10$, then what is x^a?

$$\left(x^a\right)\left(x^5\right)=x^{a+5}$$
$$\left(x^a\right)(10)=40$$
$$x^a=4$$

3. Intentionally omitted, this is a bad problem

4. If $2x+4y=20$, then what is $\frac{10-x}{2y}$?

You can plug numbers to get this, or you can do the nifty algebra trick:

$$2x+4y=20$$
$$4y=20-2x$$
$$2y=10-x$$

$$\frac{10-x}{2y}=\frac{2y}{2y}=1$$

5. If x is 400% of y, then what is $\frac{y}{x}$?

$$x = 4y$$
$$\frac{y}{x} = \frac{y}{4y} = \frac{1}{4}$$

Graphs p. 27

1. C

2. Pick the point (-2, 2) to plug into the equation:

$$y = ax^3$$
$$2 = a(-2)^3$$
$$-\frac{1}{4} = a$$

Probability p. 28

1. There are 5 green hats, 3 red hats, and 2 blue hats in a bag. What is the probability of pulling out a red hat?

$$\frac{3}{10}$$

2. There are 5 green hats, 3 red hats, and 2 blue hats in a bag. What is the probability of pulling out a red hat, a blue hat, and then a green hat? (Assume that you don't put them back in the bag after you pull them out).

$$\left(\frac{3}{10}\right)\left(\frac{2}{9}\right)\left(\frac{5}{8}\right) = \frac{1}{24}$$

Permutations p. 30

1. Mike is trying out new clothes. He has five different jeans, four different sweaters, and six different shoes. How many outfits are possible?

   5 x 4 x 6 = 120

2. Mike is creating a 3-digit code that uses the letters a, b, c, d, and e all exactly once. How many different codes are possible?

   5 x 4 x 3 = 60

3. 9 students go out for four roles in student government: President, Vice President, Secretary, and Treasurer. If each student can only take one position, how may possible arrangements of class officers are possible?

   9 x 8 x 7 x 6 = 3024

Combinations p. 31, 32

1. The probability of pulling a white T-shirt out of a laundry bag full of shirts is $\frac{2}{5}$. If there are 8 white T-shirts, how may total shirts are in the laundry bag?

$$\frac{2}{5} = \frac{8}{x}$$
$$2x = 40$$
$$x = 20$$

2. A hat is filled with 5 black, 4 green, 3 orange, and 2 purple cards. What is the probability of pulling out a green card, then an orange card, and then a purple card? Assume that you do not put the cards back in the hat once you have pulled them out.

$$\left(\frac{4}{14}\right)\left(\frac{3}{13}\right)\left(\frac{2}{12}\right) = \frac{1}{91}$$

3. A chef needs to hire one waiter, one waitress, and one hostess to form a dining staff. He has 5 waiters, 3 waitresses, and 3 hostesses to choose from. How many dining staffs are possible?

   Do not overthink this -- just a general counting problem:

$$5 \text{ x } 3 \text{ x } 3 = 45$$

4. John is filling his I Love Andy Hunt stamp collection with pleasantly awkward pictures of Andy. He has four picture frames for four separate levels of awkward. If he has three photos of level I awkward, two photos of level II awkward, 7 photos of level III awkward, and 5 photos of level IV awkward, how many possible arrangements of awkward photos are possible?

   This question is too uncomfortable to attempt to solve...Just kidding. But not really:

$$3 \text{ x } 2 \text{ x } 7 \text{ x } 5 = 210$$

5. A golf team needs to fill six of its empty spots. It has ten players to choose from. How many groups of six are possible?

$$\binom{10}{6} = 210$$

6. A swim team needs 3 new girls and 2 new boys. If it has 5 girls and 4 boys to choose from, how many teams are possible?

$$\binom{5}{3}\binom{4}{2} = 60$$

Parabolas, p. 37

1. This is a bad problem -- both B and C are correct

2. This problem should be asking for the minimum. Since you can factor it like:

$$y = x^2 - 2x + 1$$
$$y = (x-1)^2$$

3. (-2,-4) and it's a minimum

4. Assume that all the constants are positive. (B).

Functions p. 41 - 42

1. $f(x) = 3(x-2)^2$. What is $f(1) + f(2) - f(-3)$?

$$f(1) = 3(1-2)^2 = 3$$
$$f(2) = 3(2-2)^2 = 0$$
$$f(-3) = 3(-3-2)^2 = 75$$

$$f(1) + f(2) - f(-3) = 3 + 0 - 75 = -72$$

2. $f(x) = \left(\frac{x-2}{2}\right)$. What is the y-intercept of *f(x)*?

Remember that the y-intercept always happens when x =0:

$$f(0) = \left(\frac{0-2}{2}\right) = -1$$

3. $f(x) = ax^2 + bx + c$. If *a*, *b*, and *c* are positive constants, what type of graph is this? Is its y-intercept positive or negative?

This is a parabola that points upwards with a positive y-intercept.

4. C is not a function.

Tables with Functions p. 44

1. Values of $h(x)$ are given below. What is $h(-3)+h(2)$? What is $|h(-2)|-h(3)$?

x	h(x)
-3	2
-2	-4
-1	3
0	-8
1	4
2	5
3	-6

$$h(-3)+h(2)=2+5=7$$
$$|h(-2)|-h(3)=|-4|-(-6)=4+6=10$$

2. Values of $h(x)$ are given below. If $g(x)=2h(3x-1)+4$, what is g(2)?

x	h(x)
2	-2
3	-1
4	0
5	1

$$g(x)=2h(3x-1)+4$$
$$g(2)=2h(3(2)-1)+4$$
$$g(2)=2h(5)+4$$
$$g(2)=2(1)+4=6$$

3. Values of $f(x)$ are given below. If $h(x)=\frac{2f(x)}{a}$, and h(2) = -4, what is a?

x	f(x)
1	-3
2	-4
3	-2
4	-1

$$h(x)=\frac{2f(x)}{a}$$
$$h(2)=\frac{2f(2)}{a}$$
$$-4=\frac{2(-4)}{a}$$
$$a=-2$$

Graphs with Functions p. 48

1. f(x) is positive for $x < -2$ and $x > 4$. It is negative for $-2 < x < 4$.

2. B

3. The values of f(x) are shown in the table below. If g(x) = f(x) after f(x) is **reflected about** the x axis, then what is g(-2)?

   f(-2) = 6

   When we reflect the point (-2, 6), we get (-2, -6). Thus g(-2) = -6.

4. a + b = 4

Function Shifts and Scaling p. 55

1. A
2. The point labeled A has the coordinates (3,4).

   a. If $h(x) = f(x-2) + 5$, what is $h(5)$?

$$h(5) = f(5-2) + 5$$
$$h(5) = f(3) + 5$$
$$h(5) = 4 + 5 = 9$$

   b. If $h(x) = 2f(x+1) - 2$, what is $h(2)$?

$$h(2) = 2f(2+1) - 2$$
$$h(2) = 2f(3) - 2$$
$$h(2) = 2(4) - 2 = 6$$

3. B

4. The graph of $g(x)$ is shown above. If $f(x) = g(2x-1) - 3$, what is $f\left(\frac{3}{2}\right)$?

$$f\left(\frac{3}{2}\right) = g\left(2\left(\frac{3}{2}\right) - 1\right) - 3$$
$$f\left(\frac{3}{2}\right) = g(2) - 3$$
$$f\left(\frac{3}{2}\right) = 0 - 3 = -3$$

Sectors p. 61

1. A sector of a circle measures 40 degrees. If the radius of the circle is 3, what is the area of the sector? What is the arc length of this sector?

Area of the whole circle = $\pi r^2 = \pi 3^2 = 9\pi$

The sector takes up $\frac{40}{360} = \frac{1}{9}$ of the circle, so its area is $\left(\frac{1}{9}\right)9\pi = \pi$.

The circumference of the circle is $2\pi r = 2\pi(3) = 6\pi$.

The arc length of the sector is thus $\left(\frac{1}{9}\right)6\pi = \frac{2\pi}{3}$

2. The area of a sector is $\frac{1}{6}$ the area of a circle. What is the degree measure of the sector?

$$\left(\frac{1}{6}\right)360 = 60$$

3. A circle has four sectors of area a and ten sectors of area π. If the circumference of the circle is 12π, what is the value of a? What is the degree measure of each sector of length a?

$$2\pi r = 12\pi$$
$$r = 6$$

Area = $\pi r^2 = \pi 6^2 = 36\pi$

$$4a + 10\pi = 36\pi$$
$$4a = 26\pi$$
$$a = \frac{13\pi}{2}$$

Degree measure of each sector = $\left(\frac{\frac{13\pi}{2}}{36\pi}\right)360 = \left(\frac{13}{72}\right)360 = 65$

Triangles p. 64 - 66

1. This is a bad question, do not worry about this one.

2. 145

3. $\frac{3}{2}$

4. 7

5. AB

6. Another bad question, do not worry.

7. 5 + 3 + 7 = 15.

Parallel Lines p. 69

1. 215

2. 100

Number Sides, Number Angles p. 73

1. $\frac{110}{7}$

2. $\left(\frac{900}{7}\right)\frac{1}{2} = 64$

Pythagorean Thm. p. 76

1. 4

2. $2\sqrt{3}$

3. $5\sqrt{2}$

Distance Formula p. 78

1. 5

2. 13 or -11

Midpoint Problems p. 81

1. 2

2. 12

3. $\frac{6}{7}$

4. B

Shaded Regions p. 84

1. 5π

2. $\frac{32-8\pi}{32}$

3. A

Geometric Probability p. 87

1. $\frac{198-36\pi}{198}$

2. $\frac{3}{4}$

Deriving Formulas/Plugging Numbers p. 89, 90

1. 55

2. A

Odd/Even p. 92

1. C

2. E

Mean/Median/Mode p. 93 - 94

1. C

2. Bad problem, don't worry about.

Consecutive Integers p. 96

1. There is a typo here. (B) should say $4x+6$, which is the correct formula

2. E

Speed and Average Speed p. 98

1. $\frac{80}{6}$

2. $\frac{30}{7}$

Ratios p. 100

1. Typo -- should be 480 televisions, not 580. With 580 TVs, the answer is 2.

2. $\frac{1}{5}+\frac{4}{15}=\frac{7}{15}$

3. 9(6) = 54

4. 7

5. $\frac{4}{19}$

6. Bad problem, you should skip it.

Percent Change on p. 102

1. From Monday to Wednesday:

$$\frac{6-4}{4} = 50\%$$

From Tuesday to Thursday:

$$\frac{7-3}{3} = 133\%$$

2. (1.4)(0.6) = 84%